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Corollary. If $a=b=c$, $N=\frac{5}{8}a^4$, $D=a^3(1+\log 3)$.

$$\therefore M = \frac{5a}{8(1+\log 3)}. \quad M_1 = \frac{1}{3}a(1+\log 3).$$

CALCULUS.

207. Proposed by F. P. MATZ, Sc. D., Ph. D.

If K represents the complete elliptic integral of the first kind, prove that

$$\int_0^1 \frac{K d\kappa}{1+\kappa} = \frac{1}{4}\pi^2.$$

Solution by S. A. COREY, Hiteman, Iowa.

As $K = \int_0^1 \frac{dx}{\sqrt{[(1-x^2)(1-\kappa^2 x^2)]}}$, the definite integral may be written

$$\int_0^1 \int_0^1 \frac{dx d\kappa}{(1+\kappa)\sqrt{[(1-x^2)(1-\kappa^2 x^2)]}},$$

$$\text{or } \int_0^1 \frac{1}{(1-x^2)^{\frac{1}{2}}} \left[\int_0^1 \frac{d\kappa}{(1+\kappa)(1-\kappa^2 x^2)^{\frac{1}{2}}} \right] dx \dots \dots (1).$$

$$\text{If } r=\kappa+1, \int_0^1 \frac{d\kappa}{(1+\kappa)\sqrt{(1-\kappa^2 x^2)}} = \int_1^2 \frac{dr}{r\sqrt{X}} =$$

$$- \left[\frac{1}{\sqrt{a}} \log \left(\frac{\sqrt{X} + \sqrt{a}}{x} + \frac{b}{2\sqrt{a}} \right) \right]_1^2 = - \frac{1}{\sqrt{a}} \log \left(\frac{1}{1+\sqrt{a}} \right) = \frac{1}{\sqrt{a}} \log(1+\sqrt{a}),$$

in which $a=(1+x^2)$, $b=2x^2$, and $X=(1-x^2)+2x^2r-x^2r^2$.

The definite integral (1), after substituting, becomes,

$$\int_0^1 \frac{\log[1+\sqrt{(1-x^2)}]}{(1-x^2)} dx, \text{ or, if } x = \frac{1-z^2}{1+z^2}, \quad \int_0^1 \frac{1}{z} \cdot \log \left(\frac{(1+z)^2}{1+z^2} \right) dz$$

$$= 2 \int_0^1 \frac{\log(1+z)}{z} dz - \int_0^1 \frac{\log(1+z^2)}{z} dz = 2 \cdot \frac{\pi^2}{6} - \frac{\pi^2}{2 \cdot 6} = \frac{\pi^4}{4}.$$

208. Proposed by F. P. MATZ, Sc. D., Ph. D.

Solve the differential equation

$$(a^2+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0.$$

Solution by A. H. HOLMES, Brunswick, Me.

$$(a^2+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0 = \frac{d}{dx} \left[(a^2+x^2) \frac{dy}{dx} \right].$$

$$\therefore (a^2 + x^2) \frac{dy}{dx} = C; \quad dy = \frac{Cdx}{a^2 + x^2}; \quad y = \frac{C}{a} \tan^{-1} \frac{x}{a} + C'.$$

Also solved by S. A. COREY, M. E. Graber, G. W. Greenwood, J. O. Mahoney, E. L. Rich, J. Scheffer, J. E. Sanders, and G. B. M. Zerr.

GEOMETRY.

268. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find, without the aid of trigonometry, the side of an inscribed regular polygon of $2n$ sides, if the side of an inscribed regular polygon of n sides is 16 feet. [Wentworth's *Plane Geometry*, Revised Edition, problem 512, page 244.]

Solution by M. E. BECK, Cleveland, Ohio.

Let AB be the chord=16. From the center O draw the radius $r=OC$, perpendicular to AB at D . Draw the chord BC =the side of an inscribed regular polygon of $2n$ sides.

In $\triangle OBD$, $r^2 = (r - DC)^2 + 64$, and therefore $DC = r - \sqrt{(r^2 - 64)}$. Also in $\triangle CDB$, $BC^2 = DC^2 + 64$. Substituting, $BC = \sqrt{\{2r[r - \sqrt{(r^2 - 64)}]\}}$.

Also solved by P. S. Berg.

269. Proposed by J. SCHEFFER, A. M.

Find the area of a segment, if the chord of the segment is 10 feet, and the radius of the circle is 16 feet.

Solution by P. S. BERG, Larimore, N. D.

Since $16 - \sqrt{(16^2 - 5^2)} = .8014$, height of segment, then
 $\frac{(.8014)^3}{2 \times 10} + \frac{2}{3} \times .8014 \times 10 = 5.36$ square feet, area of segment.

Also solved by G. W. Greenwood, M. E. Graber, A. H. Holmes, D. B. Northrup, J. Edward Sanders, and G. B. M. Zerr.

270. Proposed by F. E. HONEY, Ph. B., Hartford, Conn.

What portion of the heavens is always invisible to an observer whose latitude is given?

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

That portion of the heavens which is never visible to an observer in a certain latitude is that cut off by the circle of perpetual occultation. Therefore, if ϕ represents the latitude of the observer, the ratio of the ever invisible portion of the celestial vault to the whole vault is manifestly $\sin^2 \frac{1}{2} \phi$.

Also solved by A. H. Holmes, and the Proposer.

271. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Two equal concentric ellipses have their axes at an angle θ . Find the area of the quadrilateral circumscribing both, in terms of θ and the semi-axes.